undetermined coefficient method

Different Try Functions to use for UDC method

① const coeff ② $G(t) = \begin{bmatrix} a \\ b \end{bmatrix}$ (polynomial, cos/sin, e^{at}) $G(t) \longrightarrow y_p$

1. $\mathbf{y}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}\mathbf{y} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{y}_0 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$ Try $\mathbf{y}_p(t) = \mathbf{a}.$ $= \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

2. $\mathbf{y}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\mathbf{y} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{y}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$ Try $\mathbf{y}_p(t) = \mathbf{a}.$

3. $\mathbf{y}' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\mathbf{y} + \begin{bmatrix} e^{-t} \\ 0 \end{bmatrix}, \quad \mathbf{y}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$ Try $\mathbf{y}_p(t) = e^{-t}\mathbf{a}.$ $= e^{-t}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

4. $\mathbf{y}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\mathbf{y} + \begin{bmatrix} e^{t} \\ -1 \end{bmatrix}, \quad \mathbf{y}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$ Try $\mathbf{y}_p(t) = e^{t}\mathbf{a} + \mathbf{b}.$ $e^t \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

5. $\mathbf{y}' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\mathbf{y} + \begin{bmatrix} t \\ -1 \end{bmatrix}, \quad \mathbf{y}_0 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$ Try $\mathbf{y}_p(t) = t\mathbf{a} + \mathbf{b}.$ $t \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

6. $\mathbf{y}' = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\mathbf{y} + \begin{bmatrix} t \\ e^{2t} \end{bmatrix}, \quad \mathbf{y}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$ Try $\mathbf{y}_p(t) = e^{2t}\mathbf{a} + t\mathbf{b} + \mathbf{c}.$

7. $\mathbf{y}' = \begin{bmatrix} -3 & -2 \\ 4 & 3 \end{bmatrix}\mathbf{y} + \begin{bmatrix} \sin t \\ 0 \end{bmatrix}, \quad \mathbf{y}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$ Try $\mathbf{y}_p(t) = (\sin t)\mathbf{a} + (\cos t)\mathbf{b}.$ $= \sin t \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \cos t \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$e^{2t}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + t\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$= \begin{bmatrix} e^{2t}a_1 + tb_1 + c_1 \\ e^{2t}a_2 + tb_2 + c_2 \end{bmatrix}$

$\begin{bmatrix} t^2 \\ 1 \end{bmatrix} \rightarrow y_p = t^2\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + t\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

To use undetermined coefficient method

- matrix A must be constant
- $G(t)$ ← polynomials, e^{at}, $\sin at / \cos at$

To use variation of parameters method

$y_p = \Phi$ where Φ is a fundamental matrix ($\omega = \det \Phi \neq 0$).

DNF?